Avoiding the Wumpus

CS 580
Intro to Artificial Intelligence

Wumpus world

Problem setup

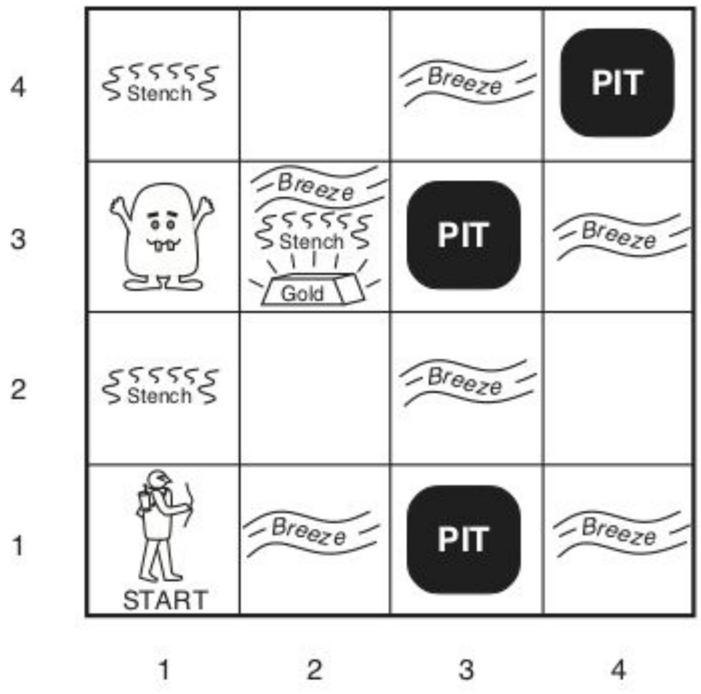
Environment

4x4 grid, containing

- 1 pile of gold
- Several pits
- Wumpus (carnivorous)

Goal

Reach the gold without falling in a pit or being eaten by the wumpus

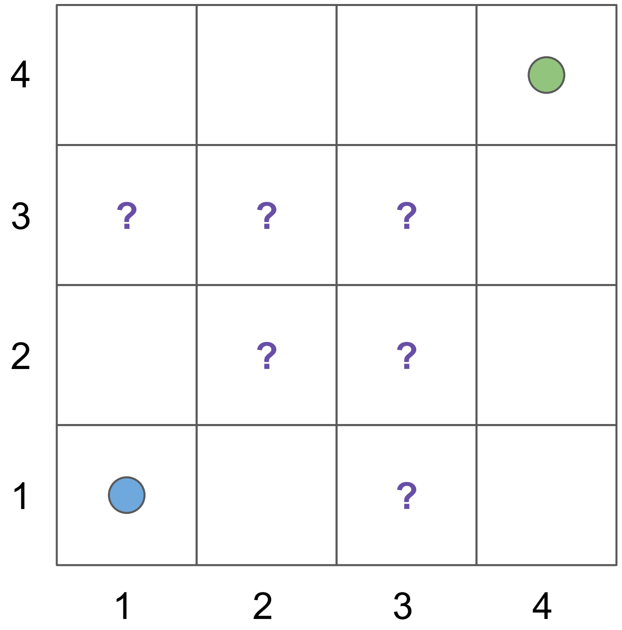


Wumpus world simplified

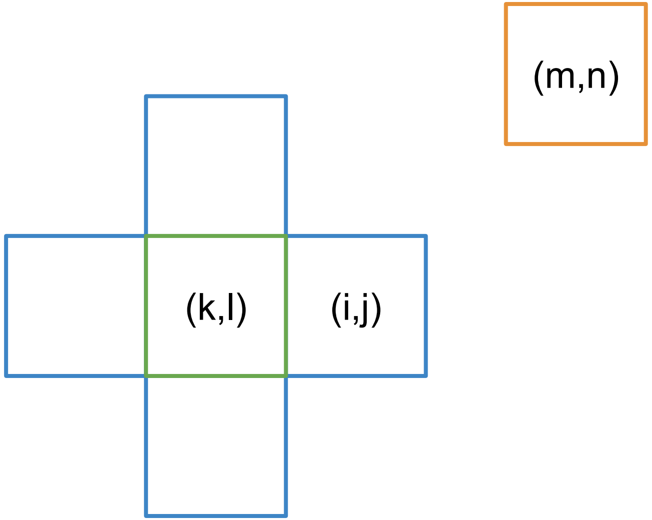
For the moment,

- Multiple Wumpuses, but they do not move
- Gold is in (4,4), agent in (1,1)
- No pits
- We have a “wumpus adjacency sensor”
- We know the marginal probability that a wumpus is in any single cell

We can use these facts to build a model of the probability that a wumpus is any specific cell, and use this to decide which cells are safest to move to.



Setting up the probabilities



Define random variables

- Let W_{ij} be the RV for the event that a wumpus is in cell (i,j)
- Let D_{kl} be the RV for the event that a wumpus is detected to be **adjacent** to cell (k,l)

Define probabilities and independence relationships

$$p(W_{ij} = \textsc{True}) = 0.2, \quad W_{ij} \perp W_{kl} \text{ for } (i,j) \neq (k,l)$$

$$p(D_{ij} = \textsc{True} \mid W_{ij+1} = \textsc{True}) = p(d_{ij} \mid w_{i+1j}) = p(d_{ij} \mid w_{ij-1}) = p(d_{ij} \mid w_{i-1j}) = 1$$

$$p(d_{ij} \mid \neg w_{ij+1}, \neg w_{i+1j}, \neg w_{ij-1}, \neg w_{i-1j}) = 0$$

$$\underline{D_{ij}} \perp \underline{W_{mn}} \mid \underline{W_{kl}} \text{ where } (i,j) \text{ adj to } (k,l)$$

Sensor is noise free, but we still can't measure wumpus direction directly

Setting up the query

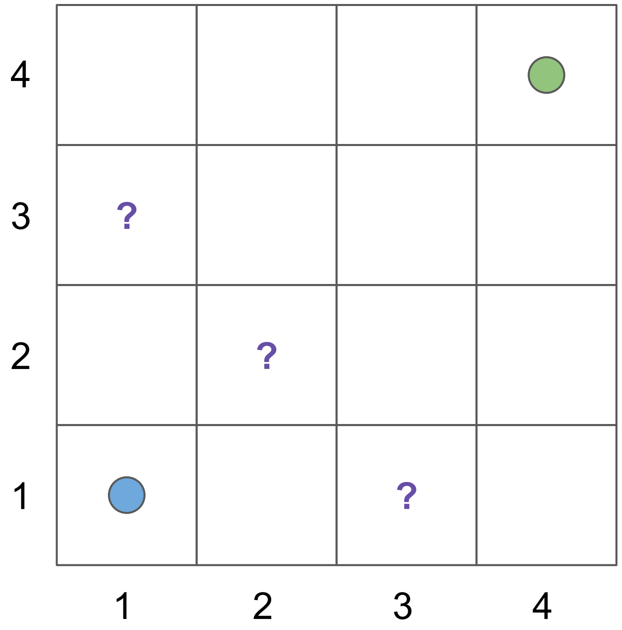
"What is the probability that a wumpus is in (1,3) given no detection in (1,1) and detections in (2,1) and (1,2)?"
We'll repeat this query for (3,1) and (2,2).

Note: if D_{11} = False, we know $W_{11} = W_{21} = W_{12} =$ False because of the sensor model

Step 1: Write down query as a probability statement

Step 2: Put into joint distribution form

Step 3: Rearrange, marginalize, plug-in



Step 1: writing the query as a probability statement

"What is the probability a wumpus is in (1,3) given no detection in (1,1), and detections in both (1,2) and (2,1)?"

$$p(W_{13} = \textsc{True} \mid D_{11} = \textsc{False}, D_{21} = \textsc{True}, D_{12} = \textsc{True},$$
$$W_{11} = \textsc{False}, W_{21} = \textsc{False}, W_{12} = \textsc{False})$$

Abbreviated version

$$p(w_{13} \mid \underline{\neg d_{11}, d_{21}, d_{12}}, \underline{\neg w_{11}, \neg w_{21}, \neg w_{12}})$$
$$p(w_{13} \mid \underline{sensors}, \underline{cleared})$$

Let "sensors" and "cleared" stand in for the collection of RV settings we started with

Step 2: Joint distribution form

“What is the probability a wumpus is in (1,3) given no detection in (1,1), and detections in both (1,2) and (2,1)?”

$$p(w_{13} \mid sensors, \ \textit{cleared} \ ) = \alpha \sum p(w_{13}, \neg d_{11}, d_{12}, d_{21}, \underline{D_{13} = \hat{d}_1, \ldots, D_{44} = \hat{d}_k,}$$

$$\neg w_{11}, \neg w_{21}, \neg w_{12}, \underline{W_{31} = \hat{w}_1, \ldots, W_{44} = \hat{w}_k)}$$

$$p(w_{13} \mid sensors, \ \textit{cleared} \ ) = \alpha \sum_h p(w_{13}, sensors, \ \textit{cleared} \ , \underline{hidden = h})$$

Here, we’re using the definition of conditional probability, normalizing, and marginalizing over all the variables not mentioned in the query

Step 3: rearrange, marginalize, normalize, solve! (1)

$$p(w_{13} \mid sensors, cleared) =$$

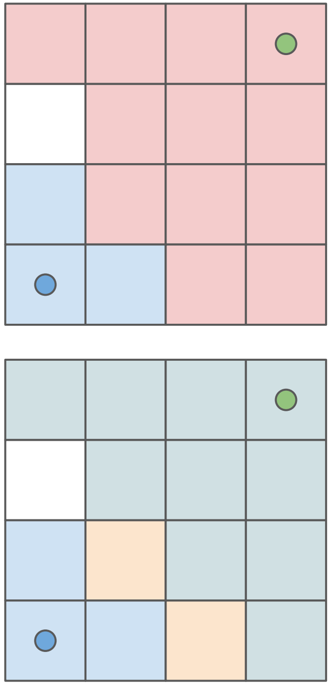
$$\alpha \sum_h p(w_{13}, sensors, \underline{cleared}, \underline{hidden = h})$$

If we split *hidden* into two pieces, *adjacent* and *far* we can leverage conditional independence to simplify

$$p(sensors \mid \underline{cleared}, \underline{adjacent}, \underline{far}) =$$

$$p(sensors \mid \underline{cleared}, \underline{adjacent})$$

We're going to have to do some rearranging first to be able to use this fact



Step 3: rearrange, marginalize, normalize, solve! (2)

$$p(w_{13} \mid sensors, cleared) = \alpha \sum_h p(w_{13}, sensors, cleared, hidden = h)$$

Break *hidden* into *adj* and *far*

$$= \alpha \sum_{h_1} \sum_{h_2} p(w_{13}, sensors, cleared, adjacent = h_1, far = h_2)$$

Use product rule

$$= \alpha \sum_{h_1} \sum_{h_2} [p(sensors \mid w_{13}, cleared, adj = h_1, far = h_2)$$
$$\cdot\, p(w_{13}, cleared, adj = h_1, far = h_2)]$$

Use conditional independence of the sensors

$$= \alpha \sum_{h_1} \sum_{h_2} \underline{p(sensors \mid w_{13}, cleared, adj = h_1)} \cdot p(w_{13}, cleared, adj = h_1, far = h_2)$$

Note: the first term does not depend on h_2

Step 3: rearrange, marginalize, normalize, solve! (3)

$$p(w_{13} \mid sensors, cleared) =$$

$$\alpha \sum_{h_1} p(sensors \mid w_{13}, cleared, adj = h_1) \left[\sum_{h_2} p(w_{13}, cleared, adj = h_1, far = h_2) \right]$$

w_{13}, *cleared*, *adjacent*, and *far* are all W_{ij} random variables, which are independent!

Don't involve h_1 or h_2

Doesn't involve h_2

$$p(w_{13} \mid sensors, cleared) =$$

$$\alpha \sum_{h_1} p(sensors \mid w_{13}, cleared, adj = h_1) \left[\sum_{h_2} p(w_{13}) \cdot p(cleared) \cdot p(adj = h_1) \cdot p(far = h_2) \right]$$

$$= \alpha \; p(w_{13}) \cdot p(\; cleared \;) \sum_{h_1} p(sensors \mid w_{13}, cleared, adj = h_1) \cdot p(adj = h_1) \sum_{h_2} p(far = h_2)$$

Step 3: rearrange, marginalize, normalize, solve! (4)

We can simplify further

$$p(w_{13} \mid sensors, cleared) =$$

$$\alpha\, p(w_{13}) \cdot \underbrace{p(\ cleared\ )}_{\text{Combine } \alpha \text{ and p(}cleared\text{)}} \sum_{h_1} \underbrace{p(sensors \mid w_{13}, cleared, adj = h_1)}_{\text{Recall the sensor model. This is either 0 or 1 depending on the values for adj}} \cdot p(adj = h_1) \overbrace{\sum_{h_2} p(far = h_2)}^{\text{Sums to 1}}$$

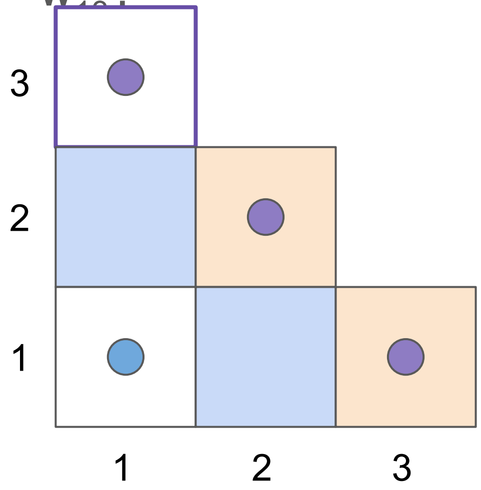
$$p(w_{13} \mid sensors, cleared) = \alpha' p(w_{13}) \sum_{h_1 \in\ cons(sensors,\ w_{13})} p(adjacent = h_1)$$

$$cons(sensors, w_{13}) = \{W_{ij} : \text{the wumpus could have generated } sensors\}$$

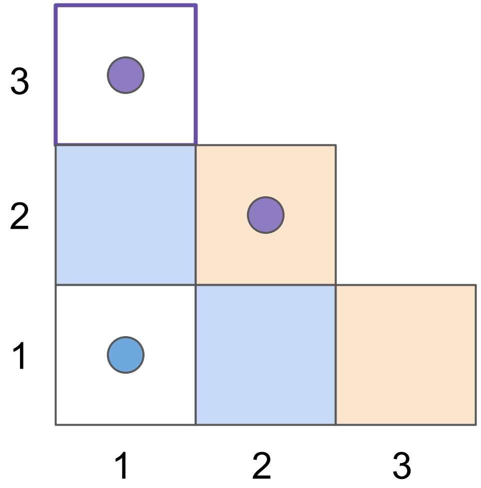
(and w_{13})

W_{ij} consistent with the sensors (1)

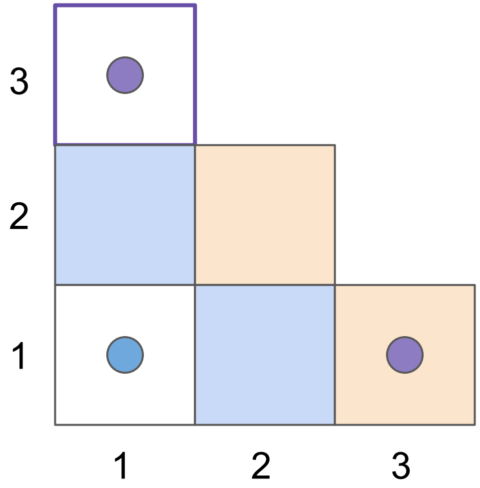
What settings for W_{ij} in *adjacent* are **consistent** with *sensors* $\neg d_{11}$, d_{12}, d_{21} and w_{13}?



p(*adj*) = $p(w_{22},w_{31}) = 0.2*0.2 = 0.04$



p(*adj*) = $p(w_{22},\neg w_{31}) = 0.2*0.8 = 0.16$

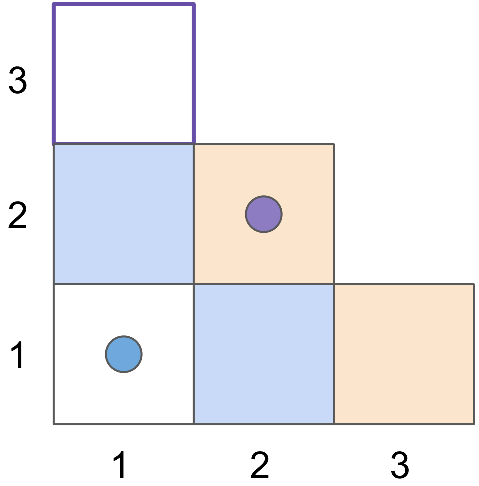


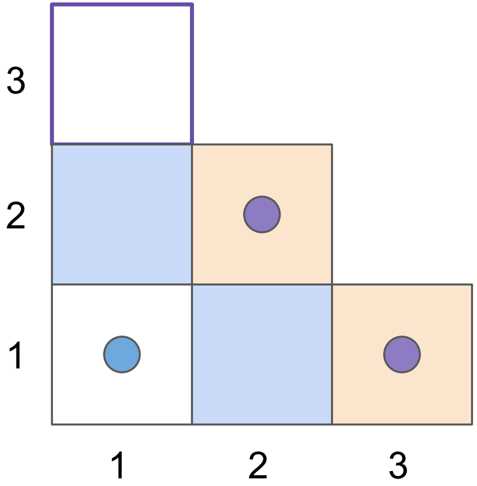
p(*adj*) = $p(\neg w_{22},w_{31}) = 0.8*0.2 = 0.16$

$p(w_{13}) = 0.2$

W_{ij} consistent with the sensors

What settings for W_{ij} in *adjacent* are **consistent** with *sensors* $\neg d_{11}$, d_{12}, d_{21} and $\neg w_{13}$?





$p(\textit{adj}) = p(w_{22}, \neg w_{31}) = 0.2*0.8 = 0.16$ $p(\textit{adj}) = p(w_{22}, w_{31}) = 0.2*0.2 = 0.04$

$p(w_{13}) = 0.8$

Solving for α

$$p(w_{13} \mid sensors, cleared) = \alpha' p(w_{13}) \sum_{h_1 \in cons(sensors,\, w_{13})} p(adjacent = h_1)$$

$$cons(sensors,\, w_{13}) = \{W_{ij} : \text{the wumpus could have generated } sensors\}$$

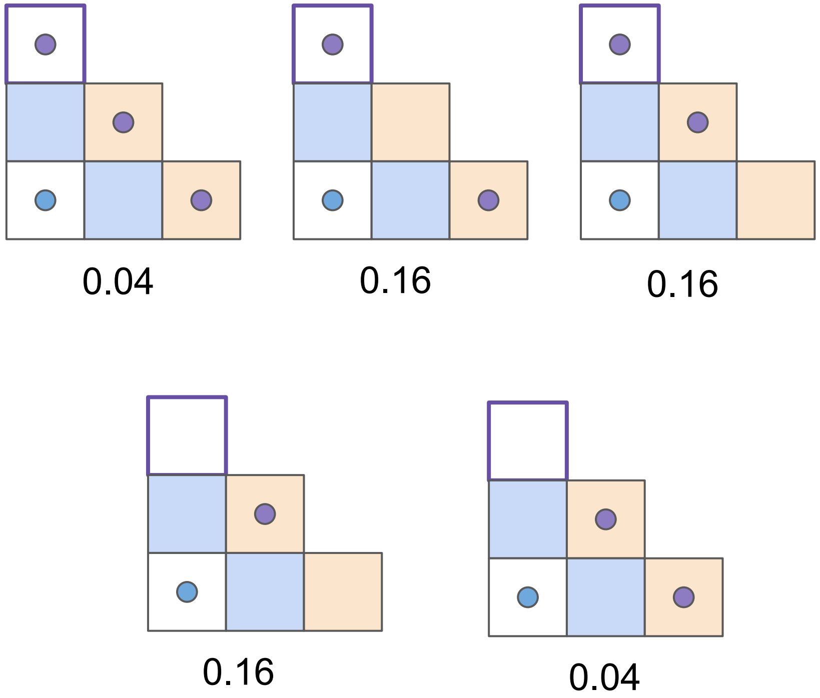
Plugging in,

$p(w_{13}|\mathit{sense},\mathit{cleared})$
$= \alpha\ p(w_{13})[p(w_{22},w_{31})+p(\neg w_{22},w_{31})+p(w_{22},\neg w_{31})]$
$= \alpha\ (0.2)\ [\ (0.04) + (0.16) + (0.16)\ ] = \alpha\ 0.072$

$p(\neg w_{13}|\mathit{sense},\mathit{cleared})$
$= \alpha\ p(\neg w_{13})[p(w_{22},\neg w_{31})+p(w_{22},w_{31})]$
$= \alpha\ (0.8)\ [\ (0.16) + (0.04)\ ] = \alpha\ 0.16$

$\alpha\ (0.072 + 0.16) = 1,\ \alpha = 4.31\ldots$

$p(w_{13}|\mathit{sense},\mathit{cleared}) = 0.310$
$p(\neg w_{13}|\mathit{sense},\mathit{cleared}) = 0.689$



Solving for w_{31} and w_{22}

W_{31} is symmetric with W_{13}, so $p(w_{31}|...)=p(w_{13}|...)=0.31$

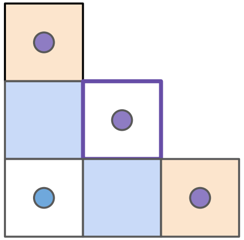
W_{22} is not symmetric, but has the same "consistent configurations" as before, just different probabilities

$p(w_{22}|\textit{sense},\textit{cleared})$
$= \alpha\ p(w_{22})[p(w_{13},w_{31})+p(w_{13},\neg w_{31})+p(\neg w_{13},w_{31})+p(\neg w_{13},\neg w_{31})]$
$= \alpha\ (0.2)\ [\ (0.04) + (0.16) + (0.16) + (0.64)\ ] = \alpha\ 0.2$

$p(\neg w_{22}|\textit{sense},\textit{cleared})$
$= \alpha\ p(\neg w_{22})[p(w_{13},w_{31})]$
$= \alpha\ (0.8)\ [\ (0.04)\ ] = \alpha\ 0.032$

$\alpha=4.31$, $p(w_{22}|\textit{sense},\textit{cleared}) = 0.86$

So the agent should avoid (2,2) for either (3,1) or (1,3)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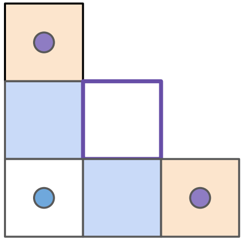
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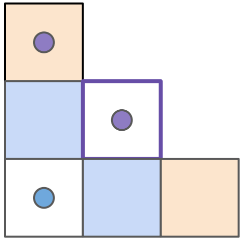
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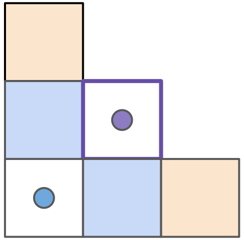
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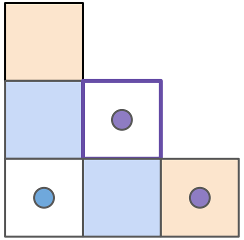
0.16

Summary and preview

Wrapping up

- We can apply all the rules of probability we've discussed to answer **queries** about the probability of specific **states** of the environment
- After developing a model, we can **compute** a desired probability with a simple three step approach
- Simplifying the model can help save significant computation overhead

Next time

- Graphically modeling independence structure using Bayes Nets